

Contents

1	Introduction	1
1.1	The Analysis of Random Experiments	1
1.2	Probability in Electrical and Computer Engineering	3
1.2.1	Signal detection and classification	3
1.2.2	Speech modeling and recognition	4
1.2.3	Coding and data transmission	4
1.2.4	Computer networks	6
1.3	Outline of the Book	6
2	The Probability Model	8
2.1	The Algebra of Events	8
2.1.1	Basic operations	9
2.1.2	Representation of the sample space	10
2.2	Probability of Events	12
2.2.1	Defining probability	12
2.2.2	Statistical independence	16
2.3	Some Applications	16
2.3.1	Repeated independent trials	17
2.3.2	Problems involving counting	18
2.3.3	Network reliability	20
2.4	Conditional Probability and Bayes' Rule	22
2.4.1	Conditional probability	22
2.4.2	Event trees	23
2.4.3	Bayes' rule	25
2.5	More Applications	26
2.5.1	The binary communication channel	26
2.5.2	Measuring information and coding	28
2.6	Summary	31
3	Random Variables and Transformations	44
3.1	Discrete Random Variables	44
3.2	Common Discrete Probability Distributions	46
3.2.1	Bernoulli random variable	47
3.2.2	Binomial random variable	47
3.2.3	Geometric random variable	48
3.2.4	Poisson random variable	50
3.2.5	Discrete uniform random variable	51
3.3	Continuous Random Variables	52
3.3.1	Probabilistic description	52
3.3.2	More about the PDF	53
3.3.3	A relation to discrete random variables	54
3.3.4	Solving problems	55
3.4	Common Continuous Probability Density Functions	57
3.4.1	Uniform random variable	57
3.4.2	Exponential random variable	58

3.4.3	Gaussian random variable	59
3.5	CDF and PDF for Discrete and Mixed Random Variables	61
3.5.1	Discrete random variables	61
3.5.2	Mixed random variables	62
3.6	Transformation of Random Variables	64
3.6.1	When the transformation is invertible	64
3.6.2	When the transformation is not invertible	67
3.6.3	When the transformation has discontinuities or flat regions . .	69
3.7	Distributions Conditioned on an Event	72
3.8	Applications	74
3.8.1	Optimal signal detection	74
3.8.2	Object classification	79
3.9	Summary	81
4	Expectation, Moments, and Generating Functions	94
4.1	Expectation of a Random Variable	94
4.1.1	Discrete random variable	95
4.1.2	Continuous random variable	95
4.1.3	Invariance of expectation	96
4.1.4	Properties of expectation	98
4.1.5	Expectation conditioned on an event	99
4.2	Moments of a Distribution	99
4.2.1	Central moments	99
4.2.2	Properties of variance	101
4.2.3	Some higher-order moments	102
4.3	Generating Functions	102
4.3.1	The moment generating function	103
4.3.2	The probability generating function	104
4.4	Application: Entropy and Source Coding	106
4.5	Summary	108
5	Two and More Random Variables	116
5.1	Two Discrete Random Variables	116
5.1.1	The joint PMF	117
5.1.2	Independent random variables	118
5.1.3	Conditional PMFs for discrete random variables	119
5.1.4	Bayes' rule for discrete random variables	120
5.2	Two Continuous Random Variables	122
5.2.1	Joint distributions	122
5.2.2	Marginal PDFs: Projections of the joint density	125
5.2.3	Conditional PDFs: Slices of the joint density	127
5.2.4	Bayes' rule for continuous random variables	130
5.3	Expectation and Correlation	132
5.3.1	Correlation and covariance	133
5.3.2	Conditional expectation	136
5.4	Gaussian Random Variables	138
5.5	Multiple Random Variables	140
5.5.1	PDFs for multiple random variables	140
5.5.2	Sums of random variables	142
5.6	Sums of Some Common Random Variables	145

5.6.1	Bernoulli random variables	145
5.6.2	Geometric random variables	146
5.6.3	Exponential random variables	146
5.6.4	Gaussian random variables	148
5.6.5	Squared Gaussian random variables	148
5.7	Random Vectors	148
5.7.1	Cumulative distribution and density functions	148
5.7.2	Expectation and moments	149
5.7.3	Multivariate Gaussian density function	151
5.7.4	Transformations of random vectors	151
5.8	An Application to Signal Detection	156
5.9	Summary	158
6	Inequalities, Limit Theorems, and Parameter Estimation	171
6.1	Inequalities	171
6.1.1	Markov inequality	172
6.1.2	Chebyshev inequality	173
6.1.3	One-sided Chebyshev inequality	174
6.1.4	Other inequalities	174
6.2	Convergence and Limit Theorems	175
6.2.1	Laws of large numbers	177
6.2.2	Central limit theorem	178
6.3	Estimation of Parameters	179
6.3.1	Estimates and properties	180
6.3.2	Sample mean and variance	181
6.4	Maximum Likelihood Estimation	183
6.5	Application to Signal Estimation	187
6.6	Summary	189
7	Random Processes	196
7.1	Random Process	196
7.1.1	The ensemble	198
7.2	First and Second Moments of a Random Process	200
7.2.1	Mean	200
7.2.2	Autocorrelation and autocovariance functions	200
7.2.3	Cross-correlation function	203
7.3	Properties: Independence, Stationarity, and Ergodicity	205
7.3.1	Statistical independence	205
7.3.2	Strict sense stationarity	205
7.3.3	Wide sense stationarity	206
7.3.4	Properties of correlation functions	210
7.3.5	Time averages and ergodic random processes	214
7.4	Power Spectral Density	217
7.4.1	Properties of the power spectral density	218
7.4.2	Cross-power spectral density	220
7.4.3	White noise	221
7.4.4	An application	227
7.5	Noise Sources	229
7.5.1	Thermal noise	230
7.5.2	Quantization noise	232

7.6	Response of Linear Systems	234
7.6.1	Linear time-invariant system	234
7.6.2	Output mean	235
7.6.3	Cross-correlation functions and cross-power spectra	236
7.6.4	Autocorrelation function and power spectral density of system output	238
7.6.5	Response of linear systems: discrete case	243
7.7	Summary	247
8	Markov and Poisson Random Processes	257
8.1	The Poisson Model	257
8.1.1	Derivation of the Poisson model	258
8.1.2	The Poisson process	260
8.1.3	An application of the Poisson process: the random telegraph signal	261
8.1.4	Additional remarks about the Poisson process	263
8.2	Discrete-Time Markov Chains	265
8.2.1	Definitions and dynamic equations	265
8.2.2	Higher-order transition probabilities	267
8.2.3	Limiting state probabilities	268
8.3	Continuous-Time Markov Chains	270
8.3.1	Simple server system	270
8.3.2	Analysis of continuous-time Markov chains	271
8.3.3	Special condition for birth and death processes	275
8.4	Basic Queueing Theory	276
8.4.1	The single-server system	277
8.4.2	Little's formula	280
8.4.3	The single-server system with finite capacity	283
8.4.4	The multi-server system	286
8.5	Summary	288
A	Basic Combinatorics	299
A.1	The Rule of Product	299
A.2	Permutations	300
A.3	Combinations	301
B	The Unit Impulse	304
B.1	The Impulse	304
B.2	Properties	306
Index		309